Mathematics Education:
A Summary of Research, Theories, and Practice

August, 2002

THOMSON NELSON™

Table of Contents

Introduction

Purpose and Origin

The purpose of this document is to summarize and share current thinking about successful approaches and strategies in mathematics education.

Mathematics instruction and resources should be based on research and grounded in what educators know about how children learn. In the course of determining what kind of elementary math resources would best meet the needs of students and educators, a host of material—including sources from Canada, the United States, New Zealand, China, Japan, and Britain—was examined. The focus was on approaches and strategies for learning and teaching that help students, in all their diversity, to achieve mathematics literacy.

Mathematics Education: A Summary of Research, Theories and Practice is an evolving document that requires ongoing revision in order to be current and useful. To that end, you are invited to add to it, comment on it, and offer suggestions for refining it. Please send your feedback to mathed@nelson.com. Please note that, because this research began as a means of informing the development of elementary classroom resources, some of the present references are less useful for those interested in secondary mathematics education.

In the Fall of 2002, a revised version will be available at www.nelson.com, with links to cited research where it is available online. Your feedback will help ensure that this document remains a "living" one, with up-to-date information about current resources and ideas—and that you will have reason to return to it often.

Background

> *Changes in society, in technology, in schools—among others—will have great impact on what will be possible in school mathematics. All of these changes will affect the fundamentals of school mathematics.*
> (Steen, 1990)

In the mid-1980s, North American educators began a movement to reform mathematics education. Partly, this reform was motivated by the factors identified (above) by Steen. The document *Everybody Counts* (National Research Council, 1989), for example, highlighted the increasing importance of mathematics to future societal growth and success. The authors recognized the need for students to be mathematically proficient in order to be prepared for a future that is ever changing and ever more reliant on mathematics. They emphasized the importance of curriculum reform and professional development, and the valuing of mathematics among parents and the public in general.

The demands of a changing world, coupled with concerns about apparent poor student performance in mathematics, have motivated reform.

Another major factor driving reform was the concern that student performance in mathematics was not at the desired level, evidence of which was documented through the results of large-scale assessments. This factor continues to motivate Canadian educators and parents. Concern about how well Canadian children are learning mathematics continues to be widespread. The research on the Third International Mathematics and Science Study (TIMSS), for instance, showed that Canadian students had trouble performing well on tasks they had been taught in the past and on questions that asked them to apply concepts and procedures to solve problems. Issues of low achievement in mathematics and poor attitudes toward learning it have been highly publicized and are a matter for continuing discussion among parents, school administrators, teachers, students, and educational publishers.

The National Council of Teachers of Mathematics (NCTM) established the Commission on Standards for School Mathematics as one way to help improve the quality of school mathematics. The NCTM was charged with creating a “coherent vision of what it means to be mathematically literate both in a world that relies on calculators and computers to carry out mathematical procedures and in a world where mathematics is rapidly growing and is extensively being applied in diverse fields” and a “set of standards to guide the revision of the school mathematics curriculum and its associated evaluation toward this vision.” (NCTM, 1989). In 2000, the NCTM published its revised *Principles and Standards for School Mathematics,* an influential document that is cited in many of these pages.

Other groups and individual educators and researchers continue to apply themselves to the improvement of mathematics education. *Mathematics Education: A Summary of Research, Theories and Practice* presents many of the results of these endeavours for your consideration.

Goals, Curricula, and Standards

The policy documents that inform school mathematics have changed across Canada. They outline not only what is taught but, to a significant extent, how it is taught. The goal is to make students mathematically literate and to enhance student achievement in mathematics.

This section starts with one description of the goal of mathematics literacy. It then examines some of the common elements in what is taught in mathematics classrooms across Canada, as outlined in provincial curricula. Finally, it provides a summary of the document *Principles and Standards for School Mathematics* (2000), developed by the National Council of Teachers of Mathematics, which is closely aligned with provincial curricula and which makes many recommendations about mathematics instruction.

Mathematics Literacy

At one time, mathematics literacy might have been defined as knowing basic number facts and having proficiency with basic skills and procedures. However, the world is rapidly changing and becoming more complex. There is an increasing need for students to understand and be able to use mathematics. In such circumstances, the old definition of mathematics literacy no longer fits.

Adding It Up

The National Research Council has produced an influential document, *Adding It Up: Helping Children Learn Mathematics,* that provides one way of describing mathematics literacy.

A mathematically literate person is described as one who demonstrates

- **Conceptual Understanding**: understanding mathematical concepts, operations, and relations
- **Procedural Fluency**: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- **Strategic Competence**: the ability to formulate, represent, and solve mathematical problems
- **Adaptive Reasoning**: the capacity for logical thought, reflection, explanation, and justification
- **Productive Disposition**: habitual inclination to see mathematics as sensible, useful, and worthwhile, combined with a belief in diligence and one's own efficiency

Mathematics Literacies:

- Conceptual Understanding
- Procedural Fluency
- Strategic Competence
- Adaptive Reasoning
- Productive Disposition

Reference:
Adding It Up: Helping Children Learn Mathematics. Washington, D.C.: National Research Council. National Academy of Science, 2001.

Provincial Mathematics Curricula

All of the provinces and territories of Canada have revised their mathematics curricula and developed new curriculum mandates.

All of the provinces and territories of Canada have revised their mathematics curricula and developed new curriculum mandates. These provincial curriculum documents have been influenced by the thinking summarized in the *Principles and Standards for School Mathematics* (2000), developed by the National Council of Teachers of Mathematics.

The western provinces have worked collaboratively to produce a mathematics curriculum through the Western Canadian Protocol (WCP). The Atlantic Provinces have also worked collaboratively to produce a common mathematics curriculum through the Atlantic Provinces Education Foundation (APEF). A number of provinces have also produced second-generation documents based on these collaborative efforts.

The curriculum of each province or territory is organized around either four or five strands in mathematics. Specific outcomes/expectations are clearly defined by grade or clusters of grades.

The following chart depicts the organizational structure of the provincial curricula for elementary mathematics:

National Council for Teachers of Mathematics	**WCP IRP (British Columbia)**	**Ontario**	**APEF**
5 Strands	**4 Strands**	**5 Strands**	**4 Strands**
Numbers and Operations	• Number Concepts • Operations	Number Sense and Numeration	• Number Concepts • Number Relationships • Operations • Number Sense
Algebra	Pattern and Relations: • Patterns • Variables and Equations • Relations and Functions	Patterning and Algebra	Patterns and Algebra
Geometry	Shape and Space: • Measurement • 3-D Objects and 2-D Shapes • Transformations	Geometry and Spatial Sense	Shape and Space: • Measurement • Geometry
Measurement	**included in Shape and Space*	Measurement	**included in Shape and Space*
Data Analysis and Probability	Statistics and Probability • Data Analysis • Chance and Uncertainty	Data Management and Probability	Data Management and Probability: • Data Management • Probability

Principles and Standards for Mathematics

Principles and Standards for School Mathematics (NCTM, 2000) identifies six basic precepts that are fundamental to excellence in mathematics education.

The National Council of Teachers of Mathematics (NCTM) is an international professional organization, based in the United States, and dedicated to excellence in teaching and learning mathematics. In 2000, NCTM published *Principles and Standards for School Mathematics.* The document had a fourfold purpose:

- to define a comprehensive set of goals for mathematics from pre-kindergarten to Grade 12
- to be a resource for educators and policy makers in analyzing and improving mathematics instruction
- to guide the development of curriculum frameworks and assessment and instructional materials
- to activate and engage in discussion at the national, provincial and local levels about how best to help students gain a deep understanding of mathematics

The Principles for School Mathematics

(from *Principles and Standards for School Mathematics*, page 11)

The principles are statements reflecting basic precepts or beliefs that are fundamental to excellence in mathematics education. The six principles are as follows:

Equity. Excellence in mathematics education requires equity—high expectations and strong support for all students.

Curriculum. A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.

Teaching. Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.

Learning. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.

Assessment. Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.

Technology. Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.

Principles and Standards for School Mathematics (NCTM, 2000) identifies a comprehensive foundation of mathematics competencies and understandings.

Standards for School Mathematics

Principles and Standards for School Mathematics identifies a comprehensive foundation of mathematics competencies and understandings for all students. The standards are descriptions of what students should know and be able to do.

Content Standards

These standards describe the content students should know:

Number and Operations
Algebra
Geometry
Measurement
Data Analysis and Probability

Process Standards

These standards describe ways of acquiring and using content knowledge:

Problem Solving
Reasoning and Proof
Communication
Connections
Representation

The 10 standards apply to all grades, from pre-kindergarten through Grade 12. However, emphasis on various standards will vary from grade to grade. For example, in primary grades, the greatest emphasis is on number and in middle grades the emphasis is on geometry and algebra. Not every topic will be addressed every year; the intent is that students will develop a level of conceptual understanding and fluency at certain points in the curriculum and then move on. Accordingly, the NCTM document also includes the standards across four different grade bands that identify these critical points in the curriculum (pre-kindergarten to Grade 2, Grades 3 to 5, Grades 6 to 8, and Grades 9 to 12).

The standards set by *Principles and Standards for School Mathematics* are:

Content Standards

Number and Operations Standard

Instructional programs from pre-kindergarten through Grade 12 should enable students to

- understand numbers, ways of representing numbers, relationships among numbers, and number systems
- understand meanings of operations and how they relate to one another
- compute fluently and make reasonable estimates

Content Standards

- Number and Operations
- Algebra
- Geometry
- Measurement
- Data Analysis and Probability

Algebra Standard

Instructional programs from pre-kindergarten through Grade 12 should enable students to

- understand patterns, relationships, and functions
- represent and analyze mathematical situations and structures using algebraic symbols
- use mathematical models to represent and understand quantitative relationships
- analyze change in various contexts

Geometry Standard

Instructional programs from pre-kindergarten through Grade 12 should enable students to

- analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
- specify locations and describe spatial relationships using coordinate geometry and other representational systems
- apply transformations and use symmetry to analyze mathematical situations
- use visualization, spatial reasoning, and geometric modelling to solve problems

Measurement Standard

Instructional programs from pre-kindergarten through Grade 12 should enable all students to

- understand measurable attributes of objects and units, systems, and processes of measurement
- apply appropriate techniques, tools, and formulas to determine measurements

Data Analysis and Probability Standard

Instructional programs from pre-kindergarten through Grade 12 should enable students to

- formulate questions that can be addressed with data and collect, organize, and display relevant data to answer questions
- select and use appropriate statistical methods to analyze data
- develop and evaluate inferences and predictions that are based on data
- understand and apply basic concepts of probability

Process Standards

Problem-Solving Standard

Instructional programs from pre-kindergarten through Grade 12 should enable students to

- build new mathematical knowledge through problem solving
- solve problems that arise in mathematics and other contexts
- apply and adapt a variety of appropriate strategies to solve problems
- monitor and reflect on the process of mathematical problem solving

Process Standards

- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representations

The standards are descriptions of what students should know and be able to do.

Reasoning and Proof Standard

Instructional programs from pre-kindergarten through Grade 12 should enable students to

- recognize reasoning and proof as fundamental aspects of mathematics
- make and investigate mathematical conjectures
- develop and evaluate mathematical arguments and proofs
- select and use various types of reasoning and methods of proof

Communication Standard

Instructional programs from pre-kindergarten through Grade 12 should enable students to

- organize and consolidate their mathematical thinking through communication
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- analyze and evaluate mathematical thinking and strategies of others
- use the language of mathematics to express mathematical ideas precisely

Connection Standard

Instructional programs from pre-kindergarten through Grade 12 should enable students to

- recognize and use connections among mathematical ideas
- understand how mathematical ideas connect and build on one another to produce a coherent whole
- recognize and apply mathematics in contexts outside of mathematics

Representation Standard

Instructional programs from pre-kindergarten to Grade 12 should enable all students to

- create and use representations to organize, record, and communicate mathematical ideas
- select, apply, and translate among mathematical representations to solve problems
- use representations to model and interpret physical, social, and mathematical phenomena

Reference:

National Council of Teachers of Mathematics. *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics, 2000.

Learning Mathematics

There has been much written about how students learn and how they can best achieve mathematics literacy. This section provides a summary of research, theories, and practice related to the following topics:

- Problem Solving
- Procedural Fluency
- Mental Math and Estimation
- Multiple Representations and Mental Imagery
- Manipulatives
- Reasoning and Reflecting
- Communicating Mathematically
- Appreciating Mathematics
- Learning Styles

Problem Solving

Student-invented algorithms enable students to generalize problem-solving steps beyond a single problem, gain confidence in their own power of reasoning, and understand the development of algorithms as creative processes.

Problem solving is central to most current definitions of mathematics literacy and to the curricula and policies that underpin mathematics education in Canada. The rationale for this emphasis is that students are seen to perform best when they are challenged to ask and answer questions, grapple with problems from a variety of sources, and think for themselves. They are better able to understand problems, plan solutions, and get correct results. Through problem solving, students can apply procedures they have learned and deepen their conceptual understanding. Problem solving is the vehicle by which students make sense of mathematics.

Given the current centrality of problem solving to mathematics education, much has been written about the importance of students discussing the nature of a problem, the strategies they use to solve the problem, and the reasons why the problem was solved in this way. In other words, if students know why a certain approach works rather than just what to do, they become better problem solvers.

Supporting theories and practice suggest that when students begin to solve mathematical problems they should be encouraged to use their own approaches to finding and recording their solutions. These student-invented algorithms enable students to generalize problem-solving steps beyond a single problem, gain confidence in their own power of reasoning, and understand the development of algorithms as creative processes greatly valued by our technological society.

Research also suggests that, in a problem-centred approach, students engage in problem solving as a process for learning about mathematical concepts and procedures. To support this approach, they need opportunities to explore ideas, and manipulate concrete objects and abstract numbers and symbols. During problem-solving activities, students perform transformations on shapes, figures, data, and graphs. They search for patterns, synthesize ideas, and draw conclusions. As they solve problems, students need time to talk and write about their understandings.

References:

- The Standards Site. National Numeracy Strategy. http://www.standards.dfee.gov.uk/numeracy
- Whitebread, D. "Children's Mathematical Thinking in the Primary Years: Perspectives on Children's Learning." In *Emergent Mathematics.* J. Anghileri (ed.). London: Cassell, 1995. 15-28.
- National Council of Teachers of Mathematics. *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics, 2000.
- Maryland State Department of Education, *Project BETTER—Building Effective Teaching Through Educational Research* http://www.mdk12.org/practices/good_instruction/projectbetter/math/general.html

Procedural Fluency

By being able to perform certain procedures automatically, without thinking about them, students are able to focus their thinking on more complex tasks.

Procedural fluency is seen as important for mathematics learning because students who have an automatic recall of number facts can use these facts as part of their repertoire of knowledge to apply to later learning. By being able to perform certain procedures automatically, without thinking about them, students are able to focus their thinking on more complex tasks.

Research cited here indicates that computational fluency comes with practice. This practice involves having students become used to working with numbers "in their heads" and using pencil and paper to keep track of their thinking. It involves engaging in activities that enable them to see the patterns that underlie the number facts (for instance, the commutative relationship between addition and subtraction; and the inverse relationship between multiplication and division). As students begin to apply patterns to their thinking, they can work for efficient and automatic speed of recall of number facts.

Basic Number Facts

If the basic number facts become a part of a child's repertoire of knowledge, these facts can readily be applied to later learning. Children need to have a strong foundation in number facts and skills in order to move to the next stage of their mathematical development.

Relate Number Facts to Number Patterns

There are 400 basic number facts, 100 for each of the four number operations. By connecting and relating these 400 facts to number patterns, students are provided with a tool that will help them to understand, learn, and remember these facts.

Children's Conceptual Understanding

Children begin to learn number facts by developing a conceptual understanding of number through the use of concrete materials, pictorial representations, and language. Through counting, students begin to make sense of number combinations. Once children are able to count, they can begin to look for number patterns. A carefully sequenced but richly woven approach to teaching and learning number facts is a proven approach with all children. Learning individual key patterns can lead, with practice, to effective mastery of between 20 or more basic facts at a time.

References:

- Heirdsfield, A. "Mental Computation: Is It More than Mental Architecture?" Presented at the annual meeting of the Australian Association for Research in Education. Sydney, Australia: December 2000. 4-7.
- Russell, Susan Jo. "Developing Computational Fluency with Whole Numbers in the Elementary Grades." In *The New England Math Journal. Millenium Focus Issue*: *Perspectives on Principles and Standards.* Beverly J. Ferrucci, and Kathleen Heid, eds. Volume XXXII, No. 2. NH: Association of Teachers of Mathematics in New England. May 2000. 40-54.
- The Standards Site. National Numeracy Strategy http://www.standards.dfee.gov.uk/numeracy

Mental Math and Estimation

Mental Math

Doing mental math builds mathematical thinking and reasoning and can make written computation easier and quicker.

Doing mental math builds mathematical thinking and reasoning and can make written computation easier and quicker. Mental calculations can lead to a better understanding of place value, mathematical operations, and basic number properties. Proficiency in mental math contributes to increased skills in estimation.

Calculating in your head is a necessary skill used in daily life. Students have the confidence and ability to rely on their own understanding and ability to carry out computations in their heads without having to rely on paper and pencil or calculators to help them arrive at an answer.

References:

- Heirdsfield, A. "Mental Computation: Is It More than Mental Architecture?" Presented at the annual meeting of the Australian Association for Research in Education. Sydney, Australia: December 2000. 4-7.
- Sowder, J. "What Are the 'Math Wars' in California All About? Reasons and Perspectives." Phi Beta Kappa Invited Lecture. Spring, 1988.

Estimation

Estimation involves fluency with computational procedures and an awareness of the kinds of calculations that can be performed easily in order to successfully estimate an answer to a problem.

The ability to estimate helps students build number sense and place-value understanding. The ability to estimate is a good indicator of students who see themselves as mathematically capable. Estimation is used far more often than paper and pencil skills in everyday life, and it is particularly important because both adults and children do more work with calculators and computers. Students need ways to check the reasonableness of answers/results.

Estimation is not a separate skill, with a set of isolated rules and techniques. Estimation involves fluency with computational procedures and an awareness of the kinds of calculations that can be performed easily in order to successfully estimate an answer to a problem.

Best practice includes encouraging students to talk about and share their ways of seeing the numbers and the operations as they estimate solutions to problems. As they do, this they develop skills to do the computation in their heads. Best practice also includes encouraging students to share thinking shortcuts.

References:

- Heirdsfield, A. "Mental Computation: Is It More than Mental Architecture?" Presented at the annual meeting of the Australian Association for Research in Education. Sydney, Australia: December 2000. 4-7.
- *Adding It Up: Helping Children Learn Mathematics.* Washington, D.C.: National Research Council. National Academy of Science, 2001.

Multiple Representations and Mental Imagery

Representations are an essential part of learning and doing mathematics. They help students organize their thinking and make mathematical ideas more concrete.

Representations are tools for thinking and communicating. Multiple representations in mathematics are used "to organize, record and communicate mathematical ideas … to solve problems … to model and interpret physical, social and mathematical phenomena" (NCTM, 2000, p. 206). Representation can be drawings, notes, tables, charts, visual organizers, or concrete manipulative materials.

Many recent studies and theories emphasize that representations are an essential part of learning and doing mathematics. They can help students organize their thinking and can help make mathematical ideas more concrete. Best practice includes encouraging students to represent their ideas in ways that make sense to them, which may mean that these early forms of representation are unconventional. It also includes, however, ensuring that students ultimately learn conventional forms of representation to ensure that they can communicate with others about mathematical ideas and that they are able to understand and use conventional representations in learning mathematics.

A summary of best practice includes the recommendations that teachers

- introduce conventional mathematical symbols, notation, equations, charts, and graphs as they connect to concepts that students are exploring and as they relate to students' developing understanding of the purpose and impact of a particular representation
- observe the circumstances in which students select and use representations to highlight the mathematical patterns and regularities that students can see
- prompt students to be flexible in choosing and creating representations: standard or non-standard, physical models or mental images that fit the purpose at hand
- model the use of representations and guide students to use different representations as they explore mathematical ideas, develop and share solutions to problems or ask questions
- ask students to identify the advantages and limitations of the various representations and evaluate representations used to present solutions to problems

References:

- Ainsworth, S., P. Bibby and D. Wood. "Analysing the Costs and Benefits of Multi-Representational Learning Environments." In *Learning with Multiple Representations.* M. van Someren, P. Reimann, H. Boshuizen, T. de Jong. Oxford: UK: Elseviere Science. 1998. 120-134.
- National Council of Teachers of Mathematics. *Principles and Standards for School Mathematics.* Reston, VA: National Council of Teachers of Mathematics, 2000.
- Van de Walle, J. *Elementary and Middle School Mathematics: Teaching Developmentally.* New York: Addison Wesley Longman, Inc. 2001.

Manipulatives give students ways to construct physical models of abstract mathematical ideas. They are also useful tools for solving problems.

Manipulatives

Many students find it useful to have and manipulate concrete representations of mathematical ideas before they understand the abstract conceptual ideas embedded in the mathematics. Manipulatives give students ways to construct physical models of abstract mathematical ideas. They are also useful tools for solving problems. In addition, manipulatives are engaging and motivating tools for students learning about mathematics.

Manipulatives are effectively used when a new concept is introduced to students and when re-teaching is necessary. Manipulatives will enhance students' understanding of the concept, though not all students may need them to be successful.

References:

- Baratta-Lorton, M. *Mathematics Their Way.* Menlo Park, CA: Addison-Wesley Publishing Company, 1976.
- D. H. & McMillen, S., "Rethinking Concrete Manipulatives." *Teaching Children Mathematics*, 2(5), 270-279. ©1996 by the National Council of Teachers of Mathematics.
- Stevenson, H.W. and J. W. Stigler. *The Learning Gap*. New York: Touchstone, 1992.

When thinking metacognitively, students are deliberately monitoring and regulating their own mathematical thinking processes. They are thinking about their thinking.

Reasoning and Reflecting

Thinking metacognitively enables students to rethink their understanding of a mathematical idea or problem. When thinking metacognitively, students are deliberately monitoring and regulating their own mathematical thinking processes. They are thinking about their thinking. Students might revise their problem-solving plan or consider using alternate problem-solving strategies; they check their calculations for accuracy and relevance and synthesize their conceptual understanding and reasoning skills.

Students learn to be metacognitive thinkers by being involved in situations where metacognitive strategies are encouraged. When children are involved in problem-solving activities and they have choices about what strategies they might use and are able to test out hypotheses, they are more likely to engage in metacognitive thinking. It is important for children to monitor their own problem-posing and problem-solving experiences. Best practice includes providing opportunities for students to reflect in writing on how and why they have used different methods and strategies.

References:

- *Adding It Up: Helping Children Learn Mathematics.* National Research Council. Washington, D.C.: National Academy of Science, 2001.
- Whitebread, D. "Children's Mathematical Thinking in the Primary Years: Perspectives on Children's Learning." In *Emergent Mathematics.* J. Anghileri (ed.). London: Cassell, 1995. 15-28.

Communicating Mathematically

Communications plays an important role in helping students construct links between their informal and intuitive notions and the abstract language and symbolism of mathematics.

Communication plays an important role in helping students construct links between their informal and intuitive notions and the abstract language and symbolism of mathematics. Different modes of mathematical communication support students in making connections among physical, pictorial, graphic, symbolic, verbal, and mental representations of mathematical ideas. When communication strategies are made explicit through class discussions, modelling, and assessment feedback, students will eventually begin to include effective active listening, paraphrasing and questioning techniques within their own mathematical conversations.

References:

- *Adding It Up: Helping Children Learn Mathematics.* National Research Council. Washington, D.C.: National Academy of Science, 2001.
- Bussi, Maria G. Bartolini. "Joint Activity in Mathematics Classrooms: A Vygotskian Analysis." In *The Culture of the Mathematics Classroom.* F. Seeger, J. Voigt and U. Waschescio, eds. Cambridge, UK: Cambridge University Press, 1998.
- National Council of Teachers of Mathematics. *Principles and Standards for School Mathematics.* Reston, VA: National Council of Teachers of Mathematics, 2000.

Appreciating Mathematics

Students with a positive attitude toward mathematics see it as both useful and worthwhile. They believe that they are capable of learning and doing mathematics. They are prepared to take risks, see problems as having many solutions, and learn from others.

Students with a positive attitude toward mathematics see it as both useful and worthwhile. They believe that they are capable of learning and doing mathematics. These students are prepared to take risks, see problems as having many solutions and learn from others. Students with a positive attitude toward mathematics take pride in their mathematical accomplishments and take an interest in things mathematical.

Students who have a positive attitude about mathematics exert more effort, spend more time on task, and are more effective learners than students with poor attitudes.

References:

- *Adding It Up: Helping Children Learn Mathematics.* National Research Council. Washington, D.C.: National Academy of Science, 2001.
- National Council of Teachers of Mathematics. *Curriculum and Evaluation: Standards for School Mathematics.* Reston, VA: National Council of Teachers of Mathematics, 1989.

Learning Styles

The term "learning style" (also referred to in the literature as "cognitive style") was first used in the 1950s but gained widespread acceptance in the 1970s. A learning style has been defined as "the method by which one comes to know or understand the world. It is the accustomed pattern used to acquire information, concepts, and skills" (Appleton, 1983). Other definitions include "… the composite of characteristic cognitive, affective and physiological factors that serve as relatively stable indicators of how a learner perceives, interacts with, and responds to the learning environment" (Keefe, 1979).

A number of learning style models have emerged and been variously embraced and discarded by educators during the past three decades. More recent approaches include Gardner's multiple intelligences model (1983) and brain-based theories of learning (Caine and Caine, 1994).

All of the theories or models with respect to learning styles place the child at the centre of the learning process.

All of the theories or models with respect to learning styles place the child at the centre of the learning process. They all emphasize that effective teaching occurs when children are given opportunities to learn in ways that maximize their strengths, while at the same time developing their less-preferred styles. All approaches stress the importance of making connections for students—connections between new learning and prior learning, between learning in one subject and learning in another, between what goes on in the classroom and the world beyond.

Four categories of learning style (Curry, 1987)

- **Personality dimensions** address issues that deal with measures of extroversion/introversion, sensing/intuition, thinking/feeling, and judging/perception (Witkin, 1954 and Myers-Briggs, 1978)

- **Information processing** considers how the individual assimilates information. Kolb's (1984) "experiential learning cycle" is the best-known model in this category. It identifies four phases of learning: concrete experience, reflective observation, abstract conceptualization, and active experimentation.

- **Social interaction** deals with the individual's interactions in the classroom. Reichmann and Grasha (1974) identified six types of learner: independent, dependent, collaborative, competitive, participant, and avoidant.

- **Instructional preference** addresses the individual's preferred learning environment. Central to this category is the model developed by Dunn and Dunn (1978), which led to the development of a Learning Styles Inventory (LSI) designed to aid educators in matching teaching environments to individual learner preferences.

Good instruction allows students not only to demonstrate their strengths but also to further develop those intelligences that are less dominant.

Cultural differences

A significant body of research has examined the extent to which learning style preferences are culturally determined. However, much of the research warns against making generalizations about the preferred learning styles of cultural groups as a whole. There exists diversity of learning style preferences in all cultures and wise use of our understanding about style preferences involves looking at each student as a unique individual.

Multiple Intelligences

Howard Gardner (1989), by defining intelligence as "the capacity to solve problems or to fashion products that are valued in one or more cultural settings," has allied himself closely with earlier learning and cognitive style theorists. Gardner's multiple intelligences model (1983) questioned the view that intelligence is limited to reason, intellect, logic, and knowledge. He has proposed that there are at least eight intelligences—and perhaps more—that include areas such as music, spatial relations, and interpersonal knowledge, as well as mathematical and linguistic intelligence. Gardner further maintains that everyone is born with these multiple intelligences but, depending on a multitude of factors, students come to school with these intelligences developed to varying degrees. He asserts that educators need to acknowledge the existence of multiple intelligences, to accept that students come to school with these intelligences developed to varying degrees, and be prepared to adjust curriculum, instruction, and assessment accordingly.

Gardner's model does not advocate simply adapting program to each student's most highly developed intelligences. Rather, good instruction allows students not only to demonstrate their strengths but also to further develop those intelligences that are less dominant.

References:

- Appleton, N. *Cultural Pluralism in Education*. New York: Longman Press, 1983.
- Caine, R. and G. Caine. *Making Connections: Teaching and the Human Brain.* Addison-Wesley/Innovative Learning Publications: Menlo Park, CA, 1994.
- Cox, B. and M. Ramirez. "Cognitive Styles: Implications for Multiethnic Education." In *Education in the 80's: Multiethnic Education*. J. Banks (ed.). Washington, D.C.: National Education Association, 1981. 61-71.
- Gardner, H. *Frames of Mind*. New York: Basic Books Inc., 1983.
- Gardner, H. *The Unschooled Mind: How Children Think and How Schools Should Teach.* New York: Basic Books Inc., 1991.
- Lazear, David. *Seven Ways of Teaching: The Artistry of Teaching with Multiple Intelligences*. Palatine, IL: IRI Skylight Publishing Inc., 1991.
- Swisher, K. and D. Deyhle. *The Styles of Learning are Different, but the Teaching is Just the Same.* Journal of American Indian Education., 1989.

ERIC Documents

- ED410029 (ERIC Document) *The Search for Style: It All Depends on Where You Look*. Tendy, Susan M. and William F. Geiser, 1997.

Teaching Mathematics

This section begins with a review of theories of and approaches to mathematics education which are currently adopted by many educators in Canada. It then summarizes a number of best practices that have been the subject of both secondary source and empirical research.

Two useful summaries of empirical research in this area are the Maryland State Department of Education's *Project BETTER — Building Effective Teaching Through Educational Research,* (available online at and the Washington State Superintendent of Instruction, Dr. Terry Bergeson's, summary *Teaching and Learning Mathematics: Using Research to Shift from the "Yesterday" Mind to the "Tomorrow Mind".*

Theories and Approaches

The teaching of mathematics today reflects the concept and the goal of mathematics literacy, but the means to achieving this goal varies. Following is a summary of some of the theories and approaches described in current research.

Constructivist Theories

Constructivist theories of teaching and learning are based on children constructing their own knowledge and conceptual understanding through their own activity.

Constructivist theories are based on the belief that children construct their own knowledge and conceptual understanding through their own activity. Piaget's theories underlie much of constructivist thought. Using Piaget's theories, it is the teacher's role to establish a mathematical environment to enable students to construct this mathematical knowledge. This environment would provide students with opportunities to hypothesize, test out their thinking, manipulate materials, and communicate their understanding in order to build mathematical knowledge.

It is the teacher's role to facilitate student learning, through setting up problems, monitoring student exploration, and negotiating meaning and understanding with the student. The teacher guides the direction of student inquiry and encourages new patterns in thinking.

Students are given a great deal of autonomy in a constructivist classroom. Pre-set lessons are not taught, since classes depend on the direction of the student's explorations.

There is wide variation of thinking among those who are proponents of a constructivist theory. This continuum ranges from radical constructivists to social constructivists, though all hold the common view that children construct their own knowledge and understanding through their own activity.

References:

- Nodding, N. "Constructivism in Mathematics Education." *Journal for Research in Mathematics Education, Monograph* #4. 1990, 7-18.
- von Glaserfeld, E. "An Exposition of Constructivism: Why Some Like It Radical." *Journal for Research in Mathematics Education, Monograph* #4. 1990, 19-29.
- Waschescio, U. "The Missing Link: Social and Cultural Aspects in Social Constructivist Theories." In *The Culture of the Mathematics Classroom*. F. Seeger, J. Voigt and U. Waschascio, (eds.). Cambridge: UK: Cambridge University Press, 1998.

Sociocultural Theory

This theory of teaching and learning is based on the work of Lev Vygotsky. He believed that there is an objective body of mathematical knowledge that comes out of the work and experience of mathematicians and constitutes the discipline of mathematics. It is this body of knowledge, mediated through the culture of the school, political institutions, and experts in the field that students need to learn and teachers need to teach. He believed that children have their own mathematical understandings and beliefs based on their experiences, but that it is the adult's responsibility to influence the child's thinking in order to move that thinking into the realm of a more scientific, conceptual understanding.

In the sociocultural theory of teaching, it is the role of the teacher to influence students' thinking, in order to move that thinking into the realm of more scientific, conceptual understanding.

Key to Vygotsky's theory was the concept that we learn from others more competent in culturally appropriate skills and technologies. Vygotsky maintained that learners, operating in the zone of proximal development (ZPD) are able to "use words and other artifacts in ways that extend beyond their current understanding of them" (Cole, Wertsch). In other words, learners are able to interact and perform actions that would be beyond their level of competence when acting alone.

The zone of proximal development is the gap between what is known and what is not known. To help guide children to attain higher levels of thinking and knowing, more capable and competent adults can explain, demonstrate, and interact with children to facilitate new learning. The ZPD is "the difference between what children or students can accomplish independently and what they can achieve in conjunction or in collaboration with another, more competent person. The zone is created in the course of social interaction" (Phillips). For Vygotsky, this interaction is with adults who display more advanced thinking.

He believed that teachers need to provide students with "conflict-generating problems" and with instruction that provides opportunities to solve problems, since this will lead to higher-level thinking and learning.

References:

- Bussi, Maria G. Bartolinie. "Joint Activity in Mathematics Classrooms: A Vygotskian Analysis." In *The Culture of the Mathematics Classroom*. F. Seeger, J. Voigt and U. Waschascio, (eds.) Cambridge: UK: Cambridge University Press, 1998.
- Cole. M. and J. V. Wertsch. *Beyond the Individual: Social Antimony in Discussions of Piaget and Vygotsky.* http://www.massey.ac.nz/~alock/virtual/colevyq.htm
- Kieran, C. "Doing and Seeing Things Differently: A 25-Year Retrospective of Mathematics Education Research on Learning." *Journal for Research in Mathematics Education*, 25, 1994, 583-607.
- Lerman, S. "A Case of Interpretations of 'Social': A Response to Steffe and Thompson." *Journal for Research in Mathematics Education*, 31, 2000, 210-227.
- Phillips, Laurie. *Primer on Topics Related to Instructional Design.* Auburn University. http://www.auburn.edu/academic/education/eflt/vyq.html
- Vygotsky, L.S. *Mind in Society*. Cambridge, MA: Harvard University Press, 1978.

Brain-Based Learning

Renate and Geoffrey Caine have evolved a theory of brain-based learning, which draws on research in neuroscience rather than cognitive psychology. Their approach, much like the approaches of their predecessors, emphasizes constructivism and active learning situations in which students are highly engaged and provided with opportunities to make connections. Caine and Caine's model includes the following principles:

- The brain is a parallel processor in that it can perform numerous activities simultaneously.
- The quality of a child's thinking depends in part on physical variables such as diet, sleep, and exercise.
- Learning is a sense-making activity in which new knowledge is acquired relative to existing knowledge.
- The brain is a pattern-seeker. New information should therefore be presented in a context, not in an isolated fashion.
- Emotions affect the quality of thinking.
- The brain processes parts and wholes simultaneously.
- Learning involves both focused attention and peripheral perception.
- Learning involves both conscious and unconscious processes.
- The brain involves multiple memory systems, including the spatial memory and the rote memory.

The brain is a pattern-seeker. New information should therefore be presented in a context, not in an isolated fashion.

Regardless of the scientific origins from which the conclusions derive, the implications for teaching and learning are clear: children learn best when they feel secure rather than threatened, when they are physically comfortable and well-nourished, when they are engaged and active rather than bored and passive, and when they see relevance and meaning in what they are learning.

References:

- Bruer, John T. "In Search of Brain-Based Education." *Phi Delta Kappan*, V. 80 No.9, May 1999.
- Caine, R. and G. Gaine. *Making Connections: Teaching and the Human Brain.* Addison-Wesley/Innovative Learning Publications: Menlo Park, CA, 1994.

Big Ideas

Students learn by making connections between new learning and what they already know. Piaget (1973) explained this process in terms of students assimilating new learning into their existing schema. Vygotsky (1978) referred to the "zone of proximal development" to describe the teachable range between what a child currently knows and the new learning that he or she is expected to acquire. Today, educators such as Van De Walle (2001), Wiggins and McTighe (1998) use terms such as "Big Ideas" and "Enduring Understandings" to describe the overarching concepts that help students truly "understand" a subject, as opposed to simply coming to know numerous discrete facts. For example, understanding that fractions and decimals are simply different ways of describing parts of a whole helps young children see the connection between the concept of fractions and the concept of decimals. Big ideas provide the "schema" that help students make sense of mathematics.

Big Ideas address the conceptual foundation of mathematics, not the procedures.

Conceived this way, big ideas make it possible for students to consider what things are similar and what things are different. Doing so helps students manage, file, organize, and connect the many concepts they are learning and thereby come to understand them more fully.

Despite increasing abstraction from grade to grade, big ideas remain fundamentally simple.

In this theory, big ideas address the conceptual foundation of mathematics, not the procedures. They are developmental insofar as they apply across all the grades. As students move from grade to grade, the level of abstraction at which the big ideas operate increases. For example, in kindergarten, children learn the concept that measurement is comparing two things. Items can be measured with another item using a similar attribute such as length. Students might measure a desk and report how many pencils wide it is. As their concepts of measurement grow and they are able to move from concrete to abstract, they begin to understand the need for people to have standard units such as centimetres.

Unlike learning a specific mathematics fact, such as 10 mm = 1 cm, students' understanding of the big ideas of mathematics does not occur at a specific moment in time. Grant Wiggins points out that big ideas have to be "uncovered" *for students*, rather than "covered" *by teachers*. For example, one big idea in measurement is that "measurement involves comparing a known quantity with one that is unknown." This big idea will not be directly taught to students at a specific moment in time; rather, students will come to this understanding over time by responding to carefully structured questions, by engaging in meaningful tasks, and by discussing the concept of measurement with their peers and the teacher.

References:

- Piaget. *To Understand Is to Invent: The Future of Education*. New York: Grossman's Publishing Co., 1973.
- Vygotsky, Lev. *Mind in Society: The Development of Higher Psychological Processes*, Cambridge, MA: Harvard University Press, 1978.
- Wiggins, G. and J. McTighe. *Understanding by Design*, Alexandria, VA: ASCD, 1998.
- Van de Walle, John. *Elementary and Middle School Mathematics: Teaching Developmentally*, 4/e. Allyn & Bacon, 2001

Comparing Three Orientations: Discovery, Transmission, and Connectionist

Discovery Orientation

Using a discovery orientation, the teacher treats all methods of calculation as equally acceptable. What is considered important is that the student obtain an answer using a method understood by the student. The effectiveness of the method is key; less attention is paid to the efficiency of the method.

This approach places great emphasis on readiness and generally interprets students' misconceptions as evidence that the students are not ready to learn these concepts.

Transmission Orientation

Using a transmission orientation, the teacher views mathematics learning as the acquisition of procedures and routines, particularly pencil and paper routines. Once these procedures have been learned, they can be applied to solving problems. There is an emphasis on standard algorithms, so students are rarely given an opportunity to discover their own methods. The focus is on the efficiency of calculation.

Comparative studies have shown that the connectionist orientation to teaching produces the most growth in student achievement in mathematics.

This approach places great emphasis on clear explanations of routines and generally interprets misconceptions as students' lack of ability or the need for reinforcement.

Connectionist Orientation

In this approach, the teacher's emphasis is on the links between different topics in the curriculum; for example, fractions, decimals, and percentages might be taught together rather than as separate topics. This approach goes far beyond investigation and problem solving and includes the use of reasoning, justification, and proof.

This approach places great emphasis on the belief that most students can learn mathematics, given appropriate teaching, which builds on students' existing strategies. However, the teacher has the responsibility to intervene to improve the efficiency of students' strategies. In a connectionist orientation, teaching mathematics is based on dialogue between the teacher and student.

Connectionist teachers make connections

- between different aspects of mathematics (e.g., showing the relationship between addition and subtraction, multiplication and division)
- between different representations of mathematics (e.g., using numbers, pictures/symbols, words, concrete objects to demonstrate a concept or procedure)
- by incorporating students' problem-solving strategies with their own approaches, thus valuing and being interested in the students' thinking

Connectionist teachers

- engage students in making connections and understanding the relationships between the many different mathematical concepts and procedures they encounter in school and in the real world
- intervene in students' learning in order to teach them the most effective and efficient way to approach specific problems
- teach students how to communicate mathematically by giving reasons for their thinking
- understand the students' thinking through talk and questioning, and help them move to a higher conceptual level
- help students view mathematics as an integrated discipline, with connections among its various strands

How to Help Students Make Connections

The National Council for Teachers of Mathematics (2000) provides a summary of what it sees as best practice for teachers who want to help students make these connections. For example, when children encounter a new mathematical concept or problem, they should be taught to ask themselves whether this idea is similar to anything they have encountered before and, if so, how it is like that previous concept or problem. By learning to make connections, students begin to see that they don't have to start from the beginning each time they learn something new. Instead, by building on what they already know and understand, they reduce the amount of new learning to be done, assimilating new ideas or integrating pieces of information into a larger, unified conceptual understanding.

References:

- The Mathematics Association. *Discover, Transmit or Connect?* Leicester, England: The Mathematics Association, 1998.
- Askew, M. It ain't (just) what you do: effective teachers of numeracy. In Thompson (Ed.) *Issues in teaching numeracy in primary schools* (pp.91-102). Buckingham, U.K. Open University Press, 1999.
- Baroody, A.J. *Fostering children's mathematical power: An investigative approach to k-8 mathematics instruction.* New Jersey: Lawrence Erlbaum Associates, 1998.
- National Council of Teachers of Mathematics. *Principles and standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics, 2000.

Best Practices

Developing Mathematical Thinking

Teachers who use the ”do-talk-record” approach promote the development of students’ skills in speaking and listening, as well as writing and reasoning. In this approach, students engage in a wide variety of activities, talk about them with each other and their teachers, and record some of whatever happens. Doing so enables them to think for themselves, form their own opinions, follow their own logic, and communicate their thinking.

References:

- Vertes, B. Doing, talking and recording with a whole class in a comprehensive school. In A. Floyd (Ed.), *Developing mathematical thinking* (pp.271-272). London: Addison-Wesley, 1981
- Maryland State Department of Education, *Project BETTER—Building Effective Teaching Through Educational Research* http://www.mdk12.org/practices/good_instruction/projectbetter/math/m-14-15.html

Attitude of Teacher and Student

While elementary school students report very definite feelings about mathematics, both negative and positive, these attitudes at early grade levels do not correlate highly with attitudes reported at later grade levels. The late elementary and early high school years appear to be an important time in the development of lasting attitudes about mathematics. In general, research indicates a decline in positive attitudes about mathematics as students progress through the grades.

All teachers can have a strong negative or positive effect on students' attitudes and achievement. Teachers who affect students positively have a strong knowledge base of the subject and demonstrate interest in the subject and the desire to have students understand mathematics.

Students' attitudes have also been found to affect the treatment they receive from teachers. Teachers seem to pay more attention to students who are sure of themselves in mathematics than to those who are less sure. This held true even if both groups of students achieved equally well.

References:

- Callahan, L. G., & Glennon, V. J. (1975). *Elementary school mathematics: A guide to current research*. Washington: Association for Supervision and Curriculum Development.
- Lindquist, M. M. (1980). Selected issues in mathematics education. Berkeley: McCutchan Publishing Corporation.
- Maryland State Department of Education, *Project BETTER—Building Effective Teaching Through Educational Research* http://www.mdk12.org/practices/good_instruction/projectbetter/math/m-27-28.html

Grouping

The way teachers group students for instruction plays a role in the learning that takes place. Teachers who group students and have short-term goals for the groups, discuss the goals with students, and regroup frequently as the goals are met, promote problem-solving skills and increase student achievement.

There are many different ways to group students for instruction. Teaching can be designed for the whole class, a small group, cooperative groups, ability groups, or for students working individually.

Grouping Depends on the Purpose of the Lesson

How children are grouped depends on the purpose of the lesson being taught. For example, a new topic or unit of work may be introduced to the whole class so that there are opportunities for all children to ask questions and discuss the concepts and ideas being presented. Following a whole class lesson, children might work in pairs or small groups or even individually to complete activities related to the lesson. Before the class ends, it is important for the teacher to bring the whole class together to review the learning and share information.

When children work in small groups, they need to have clear, effective, and sufficient instructions about the task at hand. Teachers need to monitor and work with children in their groups to ensure that any problems that have arisen can be resolved.

References:

- *Adding It Up: Helping Children Learn Mathematics*. Washington, D.C.: National Research Council. National Academy of Science, 2001.
- Stigler, J.W. and S. Hiebert. *The Teaching Gap*, New York: The Free Press, 1999.
- Driscoll, M. J. (1980). *Research within reach: Elementary school mathematics*. St. Louis: CEMREL.
- Begle, E.G. (1975). Ability grouping for mathematics instruction. *A review of the empirical literature*. (SMEGS Working Paper No. 17 ED 116 938)
- Maryland State Department of Education, *Project BETTER—Building Effective Teaching Through Educational Research* http://www.mdk12.org/practices/good_instruction/projectbetter/math/m-32.html

Review

Thought-provoking study questions, spaced throughout the instruction, seem to improve student achievement far more effectively than textbook drill pages.

Teachers who review key points or objectives after a topic is taught and who incorporate review systematically into the instructional program promote learning. Students are able to synthesize what they have learned and identify what they have not learned.

Effective review includes making outlines, questions, grouping for review, and games. Thought-provoking study questions, spaced throughout the instruction, seem to improve student achievement far more effectively than textbook drill pages. The review is not just a collection of exercises or problems; the review includes the concepts and skills that are most important to understand and remember.

Research indicates that it is better to have short periods of intensive review than long periods. Interspersing review throughout the topic is better than having extensive review at one time. Review immediately after instruction consolidates the ideas from the instruction, while delayed review aids in the re-learning of forgotten material.

Before a new topic or unit is introduced, an inventory can help the teacher ascertain whether any prerequisite knowledge needed for the topic is missing. Such a review also helps students access their prior knowledge and experiences about the topic or unit.

References:

- Suydam, M.N. (1985, May). The role of review in mathematics instruction. *Arithmetic Teacher* 33, 26.
- Good, T.L., & Grouws, D.A. (1979, June). The Missouri mathematics effectiveness project: An experimental study in fourth-grade classrooms. *Journal of Educational Psychology* 71, 355-362.
- Lee, H. (1980, July/August). The effects of review questions and review passages on transfer skills. *Journal of Educational Research* 73, 330-335.
- Maryland State Department of Education, *Project BETTER—Building Effective Teaching Through Educational Research* http://www.mdk12.org/practices/good_instruction/projectbetter/math/m-39-41.html

Using Mathematics Knowledge of Children upon Entering School

The following skills and concepts seem to be quite well developed by the time children come to school (enter Grade 1):

- rote counting by ones from 1 to 20
- identification of numerals from 1 to 10
- with objects, the concepts longest, middle, most, shortest, smallest, tallest, widest
- number combinations with objects to sums of 10
- adding 1 and 2 in verbal problems
- most facts with sums of 6 or 7
- unit fractions through halves and fourths as applied to single objects
- ordinals through sixth
- geometric figures, circle, and square
- telling time to the hour

Before children come to school, they can solve a variety of problems—usually by counting or modelling the situation with concrete objects. Students have discovered some of their mathematical ideas informally. Some of these ideas are correct, while some are not. Effective teachers help students make connections with what they already know and help them correct their misconceptions.

References:

- Callahan, L. G., & Glennon, V. J. (1975). *Elementary school mathematics*: *A guide to current research.* Washington: Association for Supervision and Curriculum Development.
- Maryland State Department of Education, *Project BETTER—Building Effective Teaching Through Educational Research* http://www.mdk12.org/practices/good_instruction/projectbetter/math/m-24-25.html

Computational Repetition

The most frequent errors with computational skills are: errors with basic facts for each operation, errors with zero for each operation, attempting to subtract the minuend from the subtrahend, adding a regrouped number later, regrouping the wrong number in multiplication, and errors with subtraction in division.

Effective teachers provide activities that help students with their specific difficulty:

- practice of basic facts for those who do not remember them
- work on place value through activities involving structured materials such as pocket charts and base 10 blocks
- work with concrete, manipulative materials for those making reversal errors

References:

- Driscoll, M. J. (1980). Research within reach: Elementary school mathematics. St. Louis: CEMREL.
- Maryland State Department of Education, *Project BETTER* http://www.mdk12.org/practices/good_instruction/projectbetter/math/m-16.html

Students learn more in classrooms where teachers increase the proportion of higher-level questions (e.g., applying, synthesizing or explaining knowledge) to lower-level questions (e.g., recall of facts and procedures).

Questioning

Skilled teachers use questions to motivate, challenge, provoke student interaction, focus on process, guide, diagnose, review, encourage exploration, invite students' questions, and enhance transfer. Students learn more in classrooms where teachers increase the proportion of higher-level questions (e.g., applying, synthesizing, or explaining knowledge) to lower-level questions (e.g., recall of facts and procedures). Asking more general questions and spending more time in classroom discussion creates more response opportunities for students.

Wait Time

The time that elapses between teachers asking for a response and eliciting a response is generally less than two seconds. Research indicates that increasing wait time to between three and five seconds can produce the following results:

For Students

- the average length of student responses increases
- the number of nonrespondents decreases
- student responses are more complex and divergent
- students initiate questions more often
- students perceive the content as less difficult
- students have more confidence in their work
- interruptions occur less frequently
- student achievement on average is higher

For Teachers

- teacher talk decreases
- teachers repeat themselves less often
- teachers repeat student responses less often
- the cognitive level of questions asked is increased
- teachers ask more probing questions

High and Low Order Questions

Teachers ask between 30 and 120 questions per hour. However, 70 percent to 95 percent of those questions are lower order, memory questions.

Effective teachers ask questions that range from low to higher order thinking skills as outlined below:

- Knowledge questions: e.g., what, when, where
- Comprehension questions: e.g., how, explain what is being asked
- Application questions: e.g., pose, demonstrate, explain how
- Analysis questions: e.g., how does it work, explain relationships
- Synthesis questions: e.g. compare, propose an alternative
- Evaluation questions: e.g., justify, verify, draw conclusions, explain your thinking

References:

- Askew, M. and D. Willam. "Effective Questioning Can Raise Achievement." *Recent Research in Mathematics Education*. London: HMSO/OFSTED. 1995. 15-16.
- Baratta-Lorton, M. *Mathematics Their Way.* Menlo Park, CA: Addison-Wesley Publishing Company. 1976.

Time Factors

Teachers have only 70% of the school day available to them for instruction.

Engaged Time

The amount of time engaged or on-task time (processing information, listening, manipulating, reading, thinking) is positively related to the amount of learning, retention over the summer, and positive attitudes about school.

Effective teachers actively engage students in mathematics instruction by giving greater attention to the lesson objectives, spending more time on presentations and discussions and less time on individual seatwork. In addition, they give encouragement frequently, minimize inappropriate behaviour, use fewer vague terms, and use relevant examples.

Instructional Time

Many factors, such as motivation, nutrition, or background affect learning. However, time on task is the single most important factor affecting learning. Research indicates, on average only about 70 percent of the time that students are in school is used for instruction and that only about 30 percent of the time that students are in school are they actually engaged on task.

The following indicates how time in the school day is typically allocated:

32 minutes daily	announcements, attendance, discipline, distributing materials
20 minutes daily	making transitions from one class, subject, or activity to another
5 minutes daily	interruptions
3 hours daily	instructional time

Research also indicates that the amount of instructional time appears to be correlated to the achievement levels of schools. In schools that are ranked as high-achieving schools, 75 percent of the school day is available for instruction. Schools that are ranked as low-achieving schools have 51 percent of the school day available for instruction.

Time on Task

Research indicates that time on task is the most important factor affecting learning. However, in elementary schools, students spend only about 32 percent of the school day on task.

Appropriate Instruction

Classrooms have a wide diversity of students—each with his or her own specific learning requirements. Yet the research states that the amount of instruction that is appropriate to each student's needs, in terms of level of difficulty, averages between 49 and 105 minutes per day.

Instruction appropriate in terms of level of difficulty averages between 49 min and 105 min per day.

Mathematics Instructional Time

The amount of time dedicated to mathematics instruction seems to have varied little over the years. The average amount of time spent on mathematics in the school day is between 30 and 40 minutes. In many primary classes, mathematics is part of an integrated day. In most classrooms beyond the primary level, mathematics is treated as a discrete subject discipline.

References:

Teddlie, C., and S. Stringield, *Schools Makes a Difference*, New York: Teachers College Press, 1993

Teddlie, C., P. Kirby and S. Stringfield, *Effective vs Ineffective Schools,* American Journal of Education, 1989.

Professional Development

The quality of instruction is a function of how well teachers know and understand the mathematics they are assigned to teach. For teachers to be effective, they must have a "profound understanding of fundamental mathematics" (Ma, 1999). By collaborating with colleagues and engaging in professional development, teachers are able to grow and develop mathematically.

Highly effective teachers perceive ongoing professional development about the mathematics they teach as having led to a major shift in their thinking.

Highly effective teachers perceive professional development as having led to a major shift in their thinking. They describe the most useful professional development as including

- discussion with other teachers
- talking to individual students in their own school as part of an assignment

Professional development at the school level increases mathematics achievement when highly effective teachers assist other teachers by working closely with them

- to plan and evaluate detailed teaching approaches
- working together in the classroom by modelling and discussing teaching approaches

Teachers perceive that, for their professional development, they require the opportunity to work individually with their mathematics coordinators over an extended period of time. They need to spend time

- attending conferences and meetings related to mathematics instruction
- working as classroom researchers
- observing, coaching, and mentoring other teachers
- trying new practices in a risk-free environment

References:

- *Effective Teachers of Numeracy*, London, England: King's College, 1999.
- *Findings on Learning to Teach*. East Lansing, MI: National Centre for Research on Teacher Learning. College of Education, Michigan State University.
- Ma, Liping. *Knowing and Teaching Elementary Mathematics* Mahwah, NJ: Lawrence Erlbaum Associates, Inc., 1999.
- National Council of Teachers of Mathematics. *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics, 2001.
- Stigler, J.W. and S. Hiebert. *The Teaching Gap*, New York: The Free Press, 1999.

Assessment in Mathematics Education

Assessment in mathematics education is currently being given a great deal of attention by educators and parents.

This section focuses on two significant areas of attention:

1) assessment and evaluation in the classroom and its relationship to student learning, instructional strategies and, ultimately, student performance

and

2) large scale assessments in Canadian elementary classrooms and what we can learn from them.

Assessment and Evaluation in the Classroom

Assessment and evaluation are hotly discussed topics in Canadian educational institutions and homes. Student achievement, measured and informed by assessment, is a high-stakes goal. Provincial curricula and policies each explicitly address these topics. This section attempts to describe assessment and evaluation in a way that is congruent with provincial policies and that draws on current, relevant theories.

In this context, assessment and evaluation can be defined as related but distinct processes. Assessment is the formal or informal gathering of information about the progress or achievement of a student, using a variety of tools and techniques. Evaluation involves judging the quality of student achievement against an accepted standard.

Assessment

Assessment serves two purposes: to inform teachers about the effectiveness of their instruction and to improve student learning. Consequently, assessment should be ongoing throughout the teaching/learning process. Depending on when it occurs, however, assessment serves different purposes, both for the teacher and students.

- **Initial or diagnostic assessment** serves two purposes: to determine what students know and can do *before* instruction begins, and to activate students' prior knowledge.
- The main purpose of **formative assessment** is to provide feedback and guidance to students *during* the learning process.
- **Summative assessment** provides opportunities for students to synthesize their learning *at the end* of a significant instructional period (a series of lessons, a unit, a chapter) and to demonstrate their learning on a broad range of learning targets.

Initial/Diagnostic Assessment

Initial or diagnostic assessment should be dynamic insofar as it is responsive to students' strengths and needs. It involves the use of a variety of assessment activities that will enable individual students and groups of students to demonstrate their current strengths and their areas of need, while at the same time activating students' prior knowledge and skills.

Formative Assessment

Formative assessment yields the greatest gains in student achievement.

Black and Wiliam reviewed 580 studies over a nine-year period. They concluded that formative assessment is one of the most significant contributors to improved learning. Five essential features of effective assessment emerged from their work:

- the provision of effective feedback to students
- the active involvement of students in their own learning
- adjusting teaching to take account of the results of assessment

- recognition of the profound influence assessment has on motivation and self-esteem
- the need for students to be able to assess themselves and understand how to improve

Wiggins and Sutton also describe assessment as involving, first and foremost, feedback to the student to inform the learning process.

Summative Assessment

Summative assessment tasks require students to synthesize a number of skills and a range of knowledge in ways that demonstrate their understanding of whatever mathematical concepts have been studied over a significant period of time. These may be summative performance tasks (sometimes called rich assessment tasks), occurring at the end of a significant period of study. Other summative tasks may be comprehensive reviews of the essential learning for a given unit of study and may take the form of more traditional written tests. As in all stages of the assessment process, it is essential that summative assessments provide students with opportunities to demonstrate their understanding through a variety of modes: written, oral, and performance-based.

Summative assessment tasks require students to synthesize a number of skills and a range of knowledge in ways that demonstrate their understanding of whatever mathematical concepts have been studied over a significant period of time.

Evaluation

Evaluation involves selecting a sufficient sample of the evidence that has been gathered through assessment and applying a judgement to it, relative to the appropriate provincial standards. Evaluation informs students, parents and school officials about how much progress has been made and/or a student’s current level of achievement. This information may be used to identify strengths and areas for improvement. Periodically, these judgements must be summarized into grades for reporting purposes. When determining report card grades in mathematics, it is neither possible nor appropriate to include all the assessment evidence that has been gathered. But, for each strand and category, teachers need to feel confident that they have sufficient data to determine a report card grade that is the best possible representation of each student’s achievement at that moment in time.

- **References:**

- Black, P. and Wiliam, D. “Inside the Black Box: Raising Standards through Classroom Assessment,” *Phi Delta Kappan*. October 1998. 139-48.
- Flewelling, G. and Higginson, W. *A Handbook on Rich Learning Tasks*. Kingston: Centre for Mathematics, Science and Technology Education. 2000.
- Guskey, T. “Reporting on Student Learning: Lessons from the Past-Prescriptions for the Future.” In *Communicating Student Learning: The ASCD Yearbook 1996*, ed. by T.R. Guskey. Alexandria, Va.: Association for Supervision and Curriculum Development.
- Stenmark, Jean Kerr (Ed.) Reston, VA: The National Council of Teachers of Mathematics. 1991.
- Sutton, R. “Assessment: a Framework for Teachers.” London: Routledge. 1992.
- Sutton, R., “Assessment for Learning in Action.” Materials presented at the Alberta Assessment Consortium Conference, Calgary. October, 2001.
- Wiggins, G. “Educative Assessment.” San Francisco: Jossey-Bass Publishers. 1998.

Large-Scale Assessments in Elementary Mathematics

Provincial Testing

On most provincial assessments, students achieved less well in problem solving and communicating.

Provincial testing authorities use a criterion-referenced approach when designing tests for large-scale assessment purposes. A criterion-referenced test measures student performance against a set of pre-determined standards. Criterion-referenced test design begins with the provincial curriculum. Test items are designed to assess learning on each outcome/expectation or on a cluster of outcomes/expectations. Performance standards are then established in the form of the targets that represent expected levels of achievement.

Provincial assessment data can help to indicate the extent to which the curriculum is being taught by teachers and learned by students. Provincial assessments provide system accountability. Provincial assessment data also serves to identify areas of the curriculum requiring attention. Provincial assessments can be used to identify areas of need in order to allocate resources.

On most provincial assessments, students achieved less well in problem solving and communicating.

National Testing: The School Achievement Indicators Program (SAIP)

National testing indicates that students do less well on problem solving in mathematics than on mathematical content.

The School Achievement Indicators Program is an initiative undertaken by the Council of Ministers of Education of Canada. SAIP was initiated in 1989 with the purpose of determining the achievement of 13- and 16-year-olds in mathematics, language, and science. The same test instrument is used with both age groups in order to determine the improvement in achievement as a result of instruction.

SAIP uses a five-level scale to measure achievement in the areas of mathematics content and problem solving. Level 1 describes the very early stages of mathematical knowledge, typical of early elementary education. Level 5 describes the knowledge and skills acquired by a student who has completed a full range of specialized mathematics courses at or near the end of secondary school.

The content component is based on the following categories:
- numbers and operations
- algebra and functions
- measurement and geometry
- data management and statistics

The problem-solving component deals with the following set of skills:
- use of numbers and symbols
- ability to reason and construct proofs
- ability to provide information and make inferences from databases

- using evaluation strategies
- communication

In 2001, 64.4% of 13 year old students achieved Level 2 or above in the content component. In the problem solving component, 67.6% of students achieved Level 2 or above. These results represent growth in achievement in both components since 1997.

Achievement of all students fell short of the expectations set by the panel of Pan-Canadian educators.

International Testing: The Third International Mathematics and Science Study Repeat (TIMSS-R 1999)

TIMMS 1999 assessed the achievement of eighth-grade students in 38 countries.

Of these countries, 26 also participated in the 1995 TIMSS. In each country, a representative sample of approximately 3500 13-14-year-olds drawn from approximately 150 schools were assessed.

The mathematics test items are based on the following five content areas:

- fractions and number sense
- measurement
- data representation, analysis, and probability
- geometry
- algebra

Countries that emphasize problem solving and reasoning achieve better results than countries favouring a more traditional approach to mathematics instruction.

Countries that emphasize problem solving and reasoning, such as Japan, achieved better results than countries that favour a more traditional approach.

Canada's mathematics performance was significantly below the international averages in the 1995 test, but similar to the international average in 1999. Ontario showed marked improvement in 1999, a result that researchers have attributed to its revised curriculum.

References:

- Alberta: http://www.learning.gov.ab.ca/k_12/testing/
- British Columbia: http://www/bced/gov.bc.ca/assessment/fsa/
- Saskatchewan: http://www.sasked.gov.sk.ca/k/pecs/ae/index.html
- Manitoba: http://www.edu.gov.mb.ca/ls4/assess/index.html
- New Brunswick: www.gov.nb.ca/education/orgs/e/eval.htm
- Newfoundland and Labrador: http://www.gov.nf.ca/edu/k12/admin.htm
- Nova Scotia: http://www.ednet.ns.ca/
- SAIP: http://www.cmec.ca/saip/indexe.stm
- TIMSS-R: http://timss.bc.edu/timss1999.html

Comparative Studies in Mathematics Education

This section focuses on studies that compare mathematics education in

1) China and the United States
2) China, Japan and the United States
3) Japan and the United States

Comparative Studies of Mathematics Education

China and the United States

Liping Ma conducted a comparative study of mathematics education in China and the United States.

Ma believes that for teachers to be effective, they must have a profound understanding of fundamental mathematics (PUFM). She indicates that the qualities needed to demonstrate this profound understanding are:

1. **Connectedness**—Teachers intentionally teach to help students see the connections among mathematical concepts and procedures. When mathematical connections are made, students begin to see mathematics as a unified body of knowledge.
2. **Multiple Perspectives**—Teachers are able to provide mathematical explanations for various approaches to a solution—whether these approaches are raised by the student or by the teacher. They are also able to articulate the advantages and disadvantages of these various facets and can lead their students to a flexible mathematical understanding.
3. **Basic Ideas**—Teachers have an underlying understanding of the basic concepts and principles of mathematics and continue to revisit and reinforce these ideas with their students.
4. **Longitudinal Coherence**—Teachers have a fundamental understanding of the whole elementary mathematics curriculum. They use it to review the concepts that students have studied before and also take opportunities to lay the proper foundation for what students will learn in the future.

For teachers to be effective they must have a profound understanding of fundamental mathematics.

Some of the key differences between the Chinese and U.S. mathematics instruction are:

- The importance in Chinese teaching of making mathematical connections among mathematical topics.
- In the Chinese system, knowledge is seen as part of a “package”. There are “key” pieces in each knowledge package. It is important to ensure that the key pieces of knowledge are taught well and that students understand them before moving to the next piece of knowledge. Key pieces provide the underlying conceptual basis for other pieces of knowledge in the package. By understanding a concept when it is first introduced, a solid basis is provided for later learning. This building of conceptual understanding does not happen in the U.S. spiral curriculum. In the United States, mastery isn’t expected the first time a topic is introduced since the same topics are revisited many times.
- The Chinese teachers believe that “learning is a continual process during which new knowledge is supported by previous knowledge and the previous knowledge is reinforced and deepened by new knowledge.”
- The Chinese teachers showed an interest in new mathematical problems and were self-confident in their ability to solve them.

In mathematics education in China, key concepts are taught and students understand them before teachers move on.

- When Chinese teachers teach a new concept, they prepare a small lecture to present to the whole class that introduces the concept or the skill. This approach to teaching trains them to talk in an organized way, unlike the U.S. teachers whose interview responses were "less mathematically relevant and mathematically organized."
- The Chinese teachers (who were not trained as mathematicians) tended to:
  - think rigorously
  - use mathematical terms to discuss a topic
  - justify their opinion with mathematical arguments
- The U.S. teachers tended to be procedurally focused. The Chinese teachers demonstrated "algorithmic competence" as well as conceptual understanding.

Reference:
Ma, Liping. *Knowing and Teaching Elementary Mathematics*. Mahway, NJ: Lawrence Erlbaum Assoc., Inc. 1999.

China, Japan, and the United States

Stevenson and Stigler observed classrooms in China, Japan, and the United States. They focused their study on instruction.

A brief summary of their findings follows:

- In U.S. schools, teachers spend less than 46 percent of their time providing instruction to students, as opposed to teachers in Japan who spend 74 percent of their time in this way.
- In U.S. schools, Grade 1 students spend more time on their own than they do in teacher-led activity.
- In the United States, students spend 47 percent of their time working individually and 10 percent in small groups.
- In Japan and China students spend most of their time in whole-class instruction.
- In Japan and China, direct instruction does not mean Socratic teaching, but rather students are engaged in the content of the lesson and lively thoughtful discussion is evident. Teachers spend little time lecturing. They ask thought-provoking questions. Students respond by generating multiple approaches to problems and by providing a rationale for their methods.
- In Japan and China, lessons begin with practical problems. The teacher leads the discussion to help students recognize what is known, what is unknown, and the critical parts of the problem. Then, students work on solving the problems, reporting on the solution, and giving their rationale. At the end of the lesson, the teacher reviews the learning.
- In China, teachers spend eight times as long summarizing the lessons as do U.S. teachers.
- In U.S. schools, the lesson often moves from one topic to another and teachers do not interrelate the components of the lesson.
- Students in Japan and China spend less time completing seatwork and worksheets than do students in the United States.
- Schools in Japan, China, and the United States use manipulatives.
- Schools in Japan and China do not use as large a variety of

manipulatives as do schools in the United States. By keeping materials consistent, teachers believe students will be able to make connections between the concepts.

- In Japan and China, students are expected to generate ideas and to evaluate the correctness of these ideas. U.S. teachers do not engage students in these kinds of discussions.

Reference:

Stevenson, H.W. and J. W. Stigler. *The Learning Gap*. New York, NY: Touchstone, 1992.

Japan and the United States

Stigler and Hiebert reviewed the TIMSS information related to the teaching of mathematics in Japan and the United States using videotapes of a sample of eighth-grade teachers in these countries. They noted that the teaching methods used by U.S. teachers, regardless of their level of competence, were limited. As a result, the ability of students to excel mathematically was also limited.

Stigler and Hiebert believe that current reforms in mathematics must also be related to improving the teaching of mathematics. Without this, they believe that the teaching gap will continue to grow, since other countries are continually improving their teaching approaches. Because the Canadian model of education is so closely linked to the American approach, the same warning may well apply here. Although there are no TIMSS videos of Canadian classrooms, many of the instructional strategies described in *The Teaching Gap* would be familiar to Canadian elementary mathematics teachers.

Teachers in the United States focus on teaching a limited band of procedural skills.

Teachers in Japan focus on teaching for conceptual understanding.

Stigler and Hiebert point out that teachers in U.S. schools focus their teaching on a limited band of procedural skills. This is the case regardless of the configuration of the classroom, or the amount or kind of manipulatives and technology available to students. Students spend most of their time acquiring isolated skills through repeated practice. Teaching in Japan is much different. In Japan, mathematics is taught in a deeper way to ensure students' conceptual understanding. In Japan, not only do students practise skills, but they also spend a great deal of time solving challenging problems and discussing mathematical concepts.

One of the variables related to effective instruction is thought to be class size, but in Canada and the United States, the average class size (using TIMSS data) is 24, while in Japan the average class size is 32 and in Korea it is 43. Despite the large class size, Korea's fourth grade students had an overall average of 76 percent in mathematics on the TIMSS assessment. Japan's overall average was 74 percent, while the U.S. average was 63 percent and Canada's average was 60 percent correct. Research on the relationship of achievement to class size points out that the main effect of smaller classes is related to teacher attitudes and instructional strategies.

Reference:

Stigler, J.W. and S. Hiebert. *The Teaching Gap*, New York: The Free Press, 1999.

Technology in Mathematics Education

The integration of calculators and Information and Communication Technologies (ICTs) in mathematics education is seen as having significant benefits by associations like the NCTM and by the developers of provincial curricula.

On the other hand, practical and pedagogical obstacles are also identified by practitioners and theorists.

This section looks at

- the outlook for the use of calculators in mathematics classrooms
- the accessibility of ICTs in Canadian Classrooms
- commonly identified benefits of their use
- and commonly identified barriers to their use

Calculators

A great deal of attention has been given to the use of calculators in mathematics classrooms. Concern is often expressed that students will become reliant on calculator use at the expense of their own ability to compute.

When calculators were used in a variety of ways, students performed as well as or better than those who used paper and pencil methods.

Many studies, using a wide variety of research methods, have been undertaken to examine the use of calculators in mathematics classrooms. The conclusions are often qualified with explanations of *how* calculators are used. When calculators were used in a variety of ways, students performed as well as or better than those who used paper and pencil methods. These studies indicate that students using calculators

- have higher math achievement than non-calculator users, even when they can choose any tool desired
- do better on mental computation than non-calculator users
- experience more varied concepts and computations
- have improved attitudes about mathematics
- do not become overly reliant on calculators

Part of being able to compute fluently means making smart choices about which tools to use and when. (NCTM, 2000)

The NCTM *Principles and Standards* document provides a model for calculator use. Teachers should encourage calculator use when:

- the focus of instruction is problem solving
- the availability of an efficient and accurate computational tool is important
- the lesson involves a search for, and an exploration of, pattern
- anxiety about the computation might hinder the problem solving
- student motivation and confidence can be enhanced through calculator use

References:

- Thompson, Anthony D. and Stephen L. Sproule. "Deciding When to Use Calculators." In *Mathematics Teaching in the Middle Grades*. NCTM. Vol. 6, #2, October 2000, 126.
- Reys, Barbara J. and Fran Arbaugh. "Clearing Up the Confusion over Calculator Use in Grades K-5." In *Teaching Children Mathematics*, NCTM, October 2001, 90-94.
- "Calculators—What Is Their Place in Mathematics Classrooms?" *Mathematics Education Dialogues*, NCTM, May/June 1999.
- National Council of Teachers of Mathematics. *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics, 2001.
- "Calculators and the Education of Youth." position statement from NCTM, 1998.
- Groves, Susie. "Calculators: A Learning Environment to Promote Number Sense." Paper presented at the annual meeting of the American Educational Research Association, New Orleans, April 1994.
- Campbell, Patricia. Project IMPACT: Increasing Mathematics Power for All Children and Teachers. Phase 1, Final Report. College Park, Md.: Center for Mathematics Education, University of Maryland, 1995.
- Sutton, John and Alice Krueger. *What we Know About Mathematics and Learning.* Mid-Continent Research, 2002.

Information & Communication Technologies (ICTs)

Based on computer to student ratio, Canada ranked first when compared with 26 other nations, as of 1999.

Computer to Student Ratio

According to the *Second International Technology in Education Study* (SITES) conducted in 1999, Canada enjoyed a better computer to student ratio than 26 other nations. In Canadian elementary schools, the computer to student ratio was one for every nine students. By province, Alberta had the best ratio (one computer for every seven students) and Nova Scotia had the fewest computers (one computer for every 15 students). In over 60 % of schools, computers are in labs.

Internet Access

In 1999, 88 percent of elementary students attended a school that has access to the Internet for instructional purposes. This figure is quite consistent with the results of SchoolNet's *On-line Connectivity Survey*, conducted in November of 1999. According to that survey

- 55% of connected computers are located in designated areas such as computer labs while 34% are located in classrooms;
- 8:1 national ratio of students per Internet connected computer; and
- 79% of schools connect via a dedicated access line

Integration of ICTs in Provincial Curricula and Policies

Every province has policy and/or curriculum related to the use of ICTs in the classroom. Some Boards have also created such documents, including scope and sequences for ICT skill development. Provincial Teacher Federations have also written policy statements.

Frequently Cited Benefits of ICT Integration

The Council of Minister of Education's 1997 report*, Developments in Information Technologies in Education,* identifies the following benefits:

- Information technologies are pedagogical tools to enhance learning and teaching.
- Students learn important new skills (computers, interactive conferencing).
- Technologies are catalysts for a revolution in the classroom since they require new approaches to learning and teaching.
- Technologies are promoting a restructuring of the curriculum with a renewed focus on the skills of accessing, managing, and processing information, collaborative working skills, problem solving and learning how to learn.
- Technologies can provide many students, especially the unmotivated, with a link between school and the real world.
- Technologies can make school relevant to learners, motivate students to greater efforts and prompt them to rethink their attitudes toward learning and schooling.
- Technologies will help students understand that almost every conceivable work possibility will require the ability to use these technologies.
- Learning is no longer bound by time and place.
- Access is provided to teachers and information beyond the school and the community.

- Lifelong learning becomes a reality.

The NCTM *Principles and Standards* document lists the following benefits, specific to mathematics education.

- With calculators and computers students can examine more examples or representational forms than are feasible by hand, so they can make and explore conjectures easily.
- The graphic power of technological tools affords access to visual models that are powerful but that many students are unable or unwilling to generate independently.
- The computational capacity of technological tools extends the range of problems accessible to students and also enables them to execute routine procedures quickly and accurately, thus allowing more time for conceptualizing and modeling.

Use of Software for Instruction

According to the *SITES*, mathematics was the subject area most likely to be taught using instructional software in all three levels of education. About 87% of elementary students, 76% of intermediate students, and 79% of secondary students attended a school that used software programs to teach math.

Barriers to Integration of ICTs

The *SITES* identified a number of obstacles related to using ICTs in the classroom. It reported on obstacles if they were seen as such by at least 50% of responding schools (measured by enrolments):

Lack of time is the most frequently cited barrier to the use of ICTs in the classroom.

- insufficient numbers of computers
- not enough types of software
- insufficient time to prepare lessons
- difficult to integrate computers into classroom
- problems scheduling computer time
- no time in teacher schedules to explore WWW
- teachers lack of ICT knowledge/skills
- not enough training opportunities

Use of ICTs and Student Achievement

A survey of findings in the cited references indicates that conclusions about what research tells us about the use of ICTs and their effect on student achievement are debated on many levels.

References:

- *Education Indicators in Canada: Report of the Pan-Canadian Education Indicator Program 1999* (Statistics Canada)
- *SchoolNet's On-line Connectivity Survey: Final Report*, April 2000.
- Developments in Information Technologies in Education. Council of Ministers of Education, 1991.
- The Power of the Internet for Learning. Web-Based Education Commission to the U.S. Congress, 2000.
- Does It Compute? The Relationship Between Educational Technology and Student Achievement in Mathematics. Educational Testing Service, 1998.
- *Taking Another Look at Technology,* (Canadian Teacher's Federations, 1997)
- Fool's Gold: A Critical Look at Computers in Childhood (Alliance for Childhood, 2000)

Home and School Connections

This section explores the importance of family involvement in mathematics education as well as the potential benefits of various kinds of homework.

Homework

It is important for teachers to differentiate homework tasks and ensure that they are appropriate to the needs of individual students.

Research related to the benefits of homework indicate that there are many benefits to students who do homework on a regular basis. These benefits have immediate as well as long-term academic and non-academic effects.

In a meta-analysis of the literature by Harris Cooper (1994), it was found that homework helps students retain information and understand material better. Homework has also been found to help students improve their critical thinking skills and can enrich the curriculum.

Cooper's research points out that homework should have different purposes for different grades. For younger students, homework should be used to foster positive attitudes and habits and help them understand that learning takes pace both inside and outside of school. Homework assignments for younger students should be short and should involve materials commonly found in the home. By middle school, homework can be used to facilitate learning in specific topics and can also include voluntary assignments that would be intrinsically interesting to these students.

Homework effects vary according to grade level, with secondary students who receive homework outperforming those who do not receive homework on standardized tests by 69 percent. Students in junior high who receive homework outperform those who do not receive homework by 35 percent. In the elementary grades, there was not an increase in achievement on standardized tests for students who did homework; however, at this level, homework is felt to be important because it promotes good study habits and positive attitudes toward school.

According to the Department of Education and Skills in the United Kingdom, it is important for teachers to differentiate homework tasks and ensure that they are appropriate to the needs of individual students. Homework should be linked as clearly as possible to classroom work and should reinforce or extend lessons and consolidate and reinforce skills and understanding. Teachers need to monitor the quality of the completed homework and mark homework on a regular basis. Homework needs to be planned and prepared alongside all other programs of learning.

Schools need to ensure they have clear policy statements regarding homework. These policies should be established in consultation with pupils, staff, and parents and should be reviewed regularly.

References:

- Cooper, Harris. "Homework Research and Policy: A review of the Literature." In *Research/Practice*, 2 (2), 1994.
- The Standards Site. Features of Good Homework Practice. Department for Education and Skills http://www.standards.dfee.gov.uk/homework/goodpractice.html

Involving Parents

Teachers can share with parents ideas for working with mathematics in children's everyday lives—sorting laundry, setting the table, or following a recipe.

When parents/guardians understand and support the school's mathematics program, they can be invaluable in convincing their children of the need to learn mathematics. Families can establish learning environments at home that enhance the work initiated at school.

Teachers need to support parents in doing mathematics with their children at home in ways that are engaging and productive. Teachers need to let parents know what is happening in class and ask parents to help ensure that homework is completed. Teachers can provide parents with lists of children's literature that have a mathematical theme or where the characters use mathematics to help them solve their problems. They can also share ideas for working with mathematics in children's everyday lives—sorting laundry, setting the table, or following a recipe.

References:

- Baratta-Lorton, M. *Mathematics Their Way.* Menlo Park, CA: Addison-Wesley Publishing Company, 1976.
- Kliman, Marlene. "Parents and Children Doing Mathematics at Home." In *Teaching Children Mathematics*. Vol. 6, #3. November 1999.
- National Council of Teachers of Mathematics. *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics, 2001.